LAU
الجامعة اللبنانية الأميركية
Lebanese American University

LAU
School of Engineering

Department of Electrical
and Computer Engineering

ELE 305: Introduction to Electrical Engineering

Final Exam – Spring 2017

Duration: **2 hours 30 minutes**
Date: 06/05/2017
Start Time: 8:00 am

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Name: ______________________________ **ID#:** ____________________

INSTRUCTIONS:

- Answer each of the following questions in the space provided.
- You can use both sides of the sheets for answers.
- Solutions written outside this booklet will not be graded.
- This is a closed-book exam
- Programmable calculators and smart devices are not allowed.
- The number of points for each question is specified next to it.
- The total number of points is 100.

1	2	3	4	5	6	7	Total
/22	/14	/8	/14	/12	/14	/16	/100

Question 1 (22 pts)

I. Convert the number 10010110 into a decimal number, if it was [6pts]
  - a. an unsigned binary number 2
  - b. a binary coded decimal 2
  - c. an 8-bit 2's complement 2

II. Convert the binary number 1011.0011 to [6pts]
  - a. A decimal number 2
  - b. An octal number 2
  - c. A hexadecimal number 2

III. Write the standard expression for f in terms of **A**, **B** and **C** for the following logic circuit [4pts]:

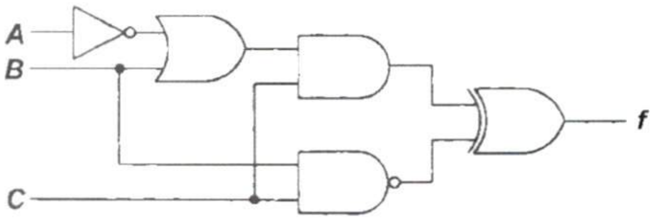


Figure 1

IV. Determine the minimum SOP expression for a function F with the following karnaugh map [6pts]:

C

AB \ CD	00	01	11	10
00	1	1	1	
01	1	1	1	
11			1	
10				

B

A

D

I - a. $(10010110)_2 = 2^7 + 2^4 + 2^2 + 2^1 = 128 + 16 + 4 + 2 = 150$

b. $(1001\ 0110)_{BCD} = 96$

c. $1001\ 0110 \xrightarrow{2\text{'s complement}} 01101010 \longrightarrow 2^6 + 2^5 + 2^3 + 2^1 = 106$

$\longrightarrow -106$

II - a. $1011.0011 = 2^3 + 2^1 + 2^0 + 2^{-3} + 2^{-4} = 11.1875$

b. $1011.0011 = 001\ 011.\ 001100 = (13.14)_8$

c. $1011.0011 = (B.3)_{16}$

III. $f = \overline{(\bar{A}+B).C}\ \overline{BC} + (\bar{A}+B).C.B.C$

$= (A\bar{B}+\bar{C})(\bar{B}+\bar{C}) + (\bar{A}+B)BC$

$= A\bar{B} + A\bar{B}\bar{C} + \bar{B}\bar{C} + \bar{C} + \bar{A}BC + BC$

$= A\bar{B}(1+\bar{C}) + \bar{C}(\bar{B}+1) + BC(\bar{A}+1)$

$= A\bar{B} + C + BC$

IV. $F = \bar{A}\bar{C} + \bar{A}D + BCD$

Question 2 (14 pts)

A game of rock paper scissors is to be implemented as a logic circuit. This hand game is played between 2 players such that each player has the option to choose rock, paper or scissors whereby rock beats scissors, scissors beats paper and paper beats rock.

The logic circuit would have two words as inputs. each word represents the choice of one player (00 if rock, 01 if paper and 10 if scissors). The output of the circuit is a single bit that is **true** if player 1 beats player 2 or both players are tied and **false** if player 2 is victorious or one of the inputs is an invalid term.

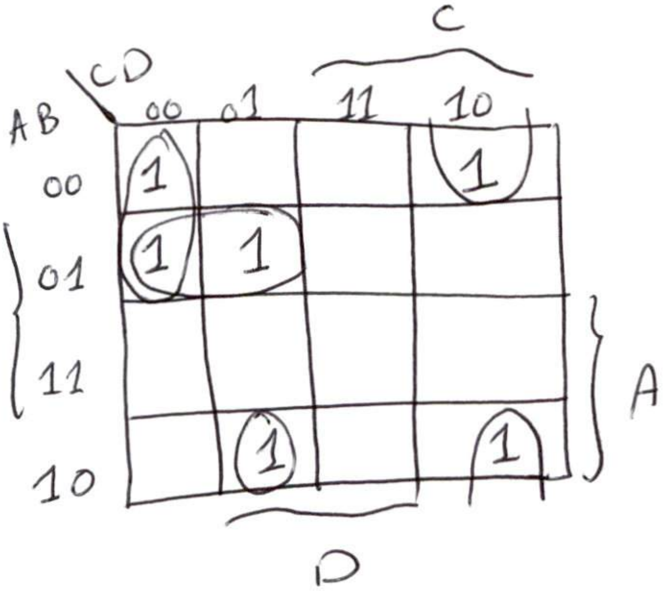
a) Build the truth table for the game including all input bits and the output. [6pts]

b) Using a karnaugh map, write a minimum SOP expression for the output in terms of the input bits. [8pts]

a)

player 1		player 2		
A	B	C	D	E
0	0	0	0	1
0	0	0	1	0
0	0	1	0	1
0	0	1	1	0
0	1	0	0	1
0	1	0	1	1
0	1	1	0	0
0	1	1	1	0
1	0	0	0	0
1	0	0	1	1
1	0	1	0	1
1	0	1	1	0
1	1	0	0	0
1	1	0	1	0
1	1	1	0	0
1	1	1	1	0

c)



AB \ CD	00	01	11	10
00	1			1
01	1	1		
11				
10		1		1

$$E = \overline{B}C\overline{D} + \overline{A}\,\overline{C}\,\overline{D} + \overline{A}B\overline{C} + A\overline{B}\,\overline{C}D$$

Question 3 (8 pts)

The energy consumed by the source in Figure 2.a is given by the waveform in Figure 2.b.

a) Calculate and sketch the current i(t) [4pts]

b) Calculate the average power consumed by the box over the period t=0 to 8ms [4pts]

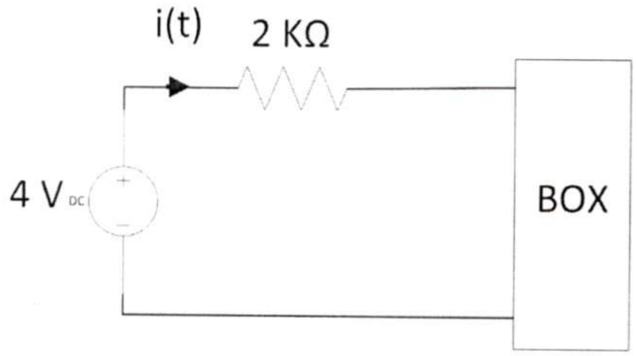


Figure 2.a

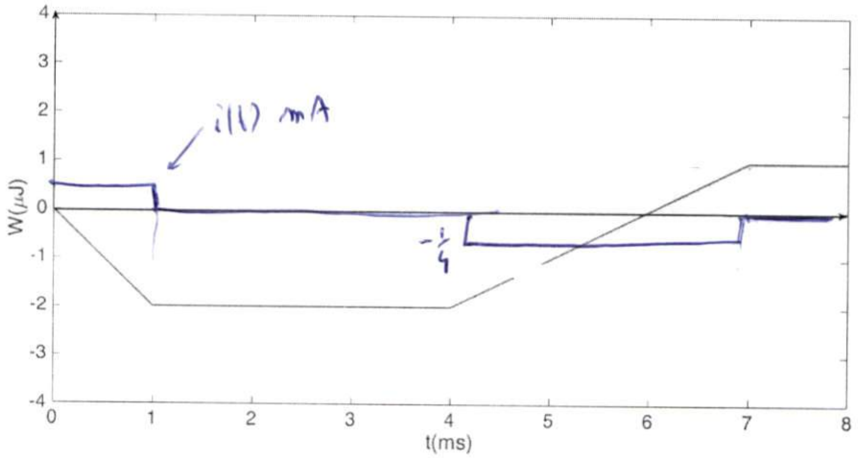


Figure 2.b

a) $\overset{mW}{P} = -V\,i = \dfrac{dw(t)^{\,\mu J}}{dt_{\to ms}}$

$$\Rightarrow \overset{mA}{i} = -\frac{1}{V}\frac{dw}{dt} = + \begin{cases} -\frac{1}{4} \times -2 = \frac{1}{2} & 0 \leqslant t \leqslant 1 \\ 0 & 1 \leqslant t \leqslant 4 \\ -\frac{1}{4} \times 1 = -\frac{1}{4} & 4 \leqslant t \leqslant 7 \\ 0 & 7 \leqslant t \leq 8 \end{cases}$$

b) $P_{avg,source} = \frac{1}{8}\int_0^8 p(t)\,dt$

$= \frac{1}{8}\int_0^8 v\,i\,dt = \frac{1}{2}\int_0^8 i(t)\,dt$

$= \frac{1}{2}\left[\int_0^1 \frac{1}{2}\,dt + \int_4^7 -\,dt\right] = \frac{1}{2}\left[\frac{1}{2} - (7-4)\right]$

$= \frac{1}{2}\left[\frac{1}{2} - 3\right] = -\frac{5}{4}$ mW

$P_{avg,2k\Omega} = \frac{1}{8}\int_0^8 R i^2\,dt$

$= \frac{1}{8}\left[\int_0^1 2\times\frac{1}{4}\,dt + \int_4^7 2\times\frac{1}{16}\,dt\right]$

$= \frac{1}{8}\left[\frac{1}{2} + \frac{3}{8}\right] = \frac{1}{8}\left[\frac{7}{8}\right] = \frac{7}{64}$ mW

$P_{avg,box} = P_{avg,source\ supplied} - P_{avg,2k\Omega\ absorbed}$

$= \frac{5}{4} - \frac{7}{64} = \frac{80-7}{64} = \frac{73}{64}$ mW

Question 4 (14 pts)

The input to the network shown in Figure 3.a is given by the waveform in Figure 3.b. The operational amplifier is considered ideal.

a) Find an expression of $v_o(t)$ in terms of $v_i(t)$. [10pts]
b) Draw, on the same graph of Figure 3.b., the output waveform $v_o(t)$. [4pts]

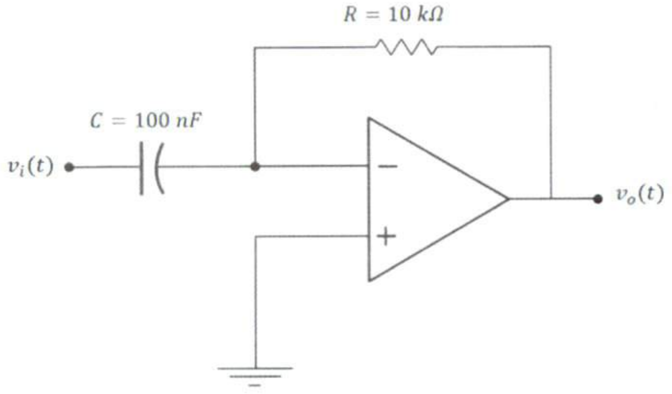


Figure 3.a

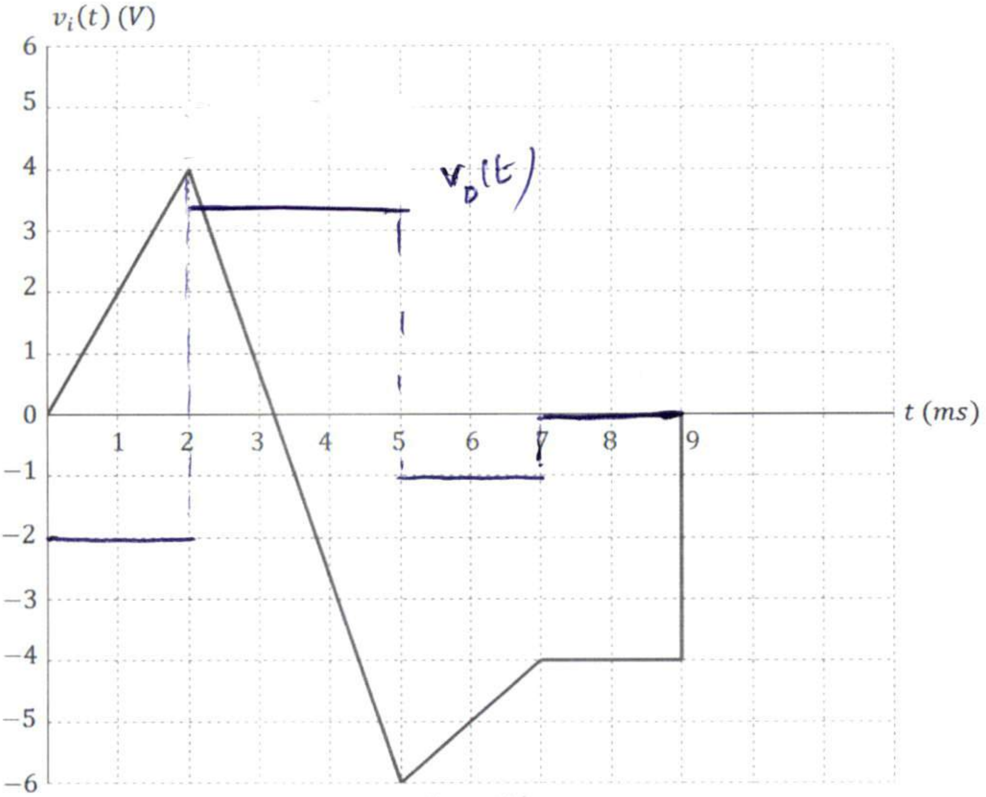


Figure 3.b

a - $V_+ = V_- = 0$

$$C\frac{dv_i}{dt} = -\frac{v_o}{R} \Rightarrow v_o = -RC\frac{dv_i}{dt}$$

$$= -10^{-3}\frac{dv_i}{dt} = -\frac{\overset{v}{dv_i}}{\underset{ms}{dt}}\ (v)$$

Question 5 (12 pts)

Consider the circuit in Figure 3.

a) Find an expression for I using superposition. [10pts]

b) Suppose $R_2 I_1 = R_4 I_2$. Is the voltage source Vo absorbing or supplying power? [2pts]

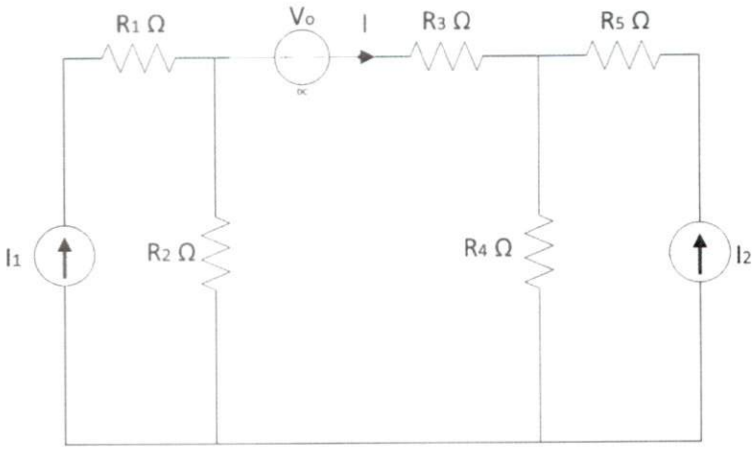
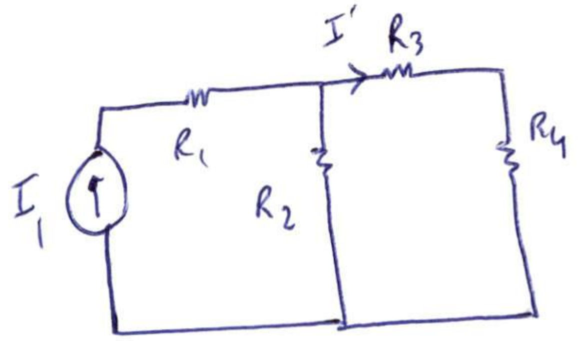


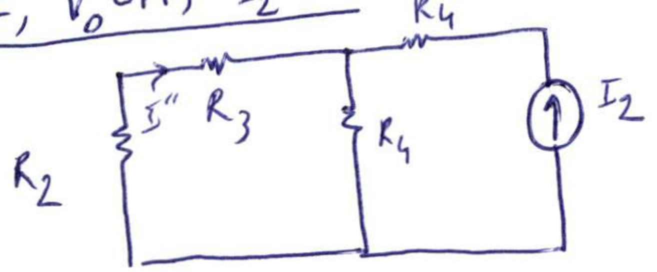
Figure 3

I_1 ON, V_0 OFF, I_2 OFF



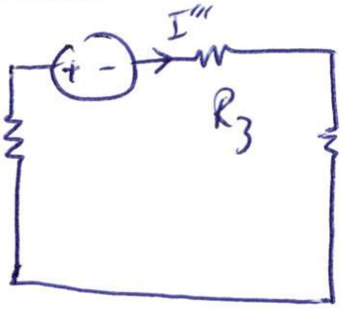
Current divider: $I' = I_1 \times \frac{R_2}{R_2 + R_3 + R_4}$

I_1 OFF, V_0 OFF, I_2 ON



Current divider: $I'' = -I_2 \frac{R_4}{R_2 + R_3 + R_4}$

I_1 OFF, V_2 ON, I_2 OFF



KVL: $V_0 = -I'''(R_2+R_3+R_4)$

$$\Rightarrow I''' = \frac{-V_0}{R_2+R_3+R_4}$$

Finally: $I = I' + I'' + I'''$

$$= \frac{1}{R_2+R_3+R_4}(R_2 I_1 - R_4 I_2 - V_0)$$

b) $R_2 I_1 = R_4 I_2 \Rightarrow I = \frac{-V_0}{R_2+R_3+R_4}$

power of V_0: $P_0 = +V_0 I$

$$= \frac{-V_0^2}{R_2+R_3+R_4} < 0 \Rightarrow \text{supplying}$$

Question 6 (14 pts)

Find the thevenin equivalent of the circuit in Figure 4 as seen between points A and B. $V_s=1<180^o$ and $I_s=2<-90^o$

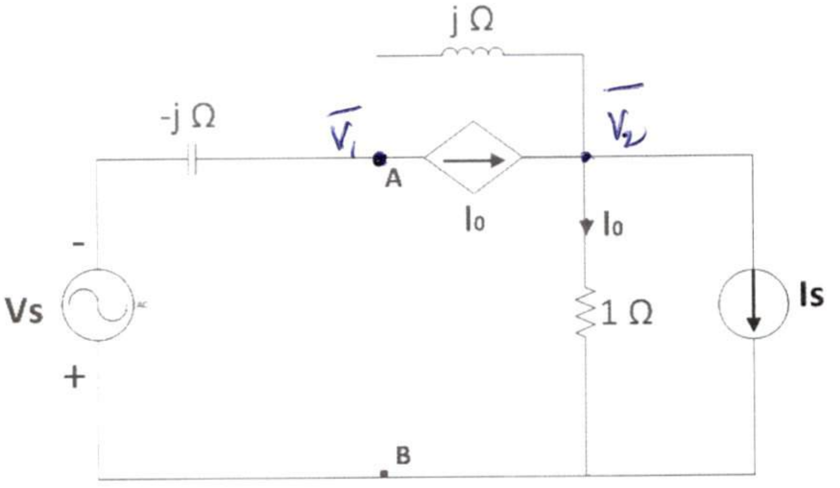


Figure 4

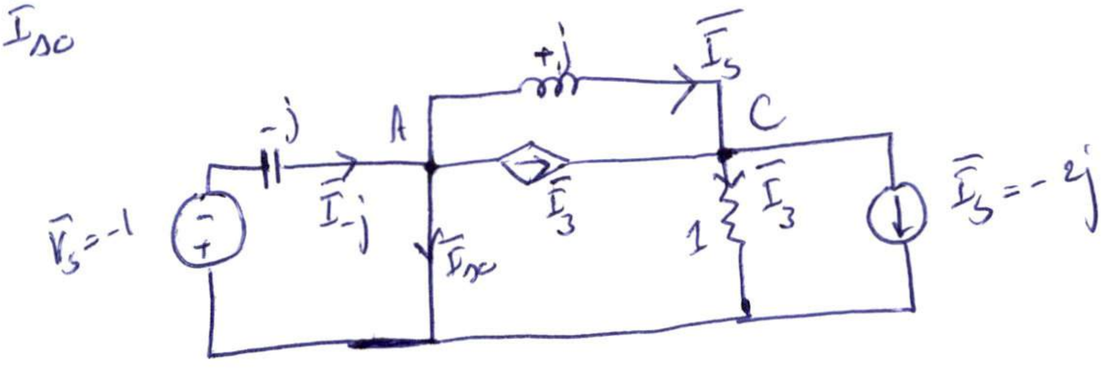
Ⓘ $\bar{V}_{th} = \bar{V}_{AB} = \bar{V}_1$

$$\left\{\begin{array}{l} \dfrac{\bar{V}_1+\bar{V}_s}{-j} + \dfrac{\bar{V}_1-\bar{V}_2}{j} + \bar{I}_0 = 0 \\ \dfrac{\bar{V}_2-\bar{V}_1}{j} - \bar{I}_3 + \bar{V}_2 + \bar{I}_s = 0 \\ \text{and } \bar{V}_2 = \bar{I}_0 \end{array}\right\} \Rightarrow \begin{array}{l} \dfrac{\bar{V}_1-1}{-j} + \dfrac{\bar{V}_1-\bar{V}_2}{j} + \bar{V}_2 = 0 \quad ① \\ \dfrac{\bar{V}_2-\bar{V}_1}{j} + \bar{I}_s = 0 \quad ② \end{array}$$

$$① \Rightarrow j\bar{V}_1 - j - j\bar{V}_1 + j\bar{V}_2 - \bar{V}_2 = 0 \Rightarrow \bar{V}_2 = \frac{1+j}{2}$$

$$② \Rightarrow \bar{V}_2 - \bar{V}_1 = -j\bar{I}_s \Rightarrow \bar{V}_1 = \bar{V}_2 + 2 = \frac{5+j}{2} = \overline{V_{th}}$$

Ⅱ $\overline{I}_{sc}$



KCL@C: $\overline{I}_{+j} = \overline{I}_S = -2j$

Ohm's law: $\overline{I}_{-j} = -\overline{V}_S \times (+j) = +j$

Ohm's law: $\overline{I}_3 = \overline{V}_C - \overline{V}_A$
$= \overline{V}_{CA} = -j\,\overline{I}_{AC} = -j(-2j) = -2$

Finally KCL@A: $\overline{I}_{-j} = \overline{I}_{sc} + \overline{I}_{+j} + \overline{I}_3$

$\Rightarrow \overline{I}_{sc} = 2+3j$

and $Z_{th} = \frac{\overline{V}_{oc}}{\overline{I}_{sc}} = \frac{5+j}{2(2+3j)} =$ [illegible] $\frac{1-j}{2}$

Question 7 (16 pts)

A 480 $V\ rms$, 60 Hz voltage source delivers power to three loads in parallel:

- Load A: Purely resistive load of 4 kW.
- Load B: inductive load. 28 kVA at 0.707 power factor.
- Load C: Capacitive load. 2 kW and 2 $kVar$.

a. Find the total power, the total reactive power, the power factor for the source, and the rms source current. [10pts]

b. A fourth load that is purely capacitive is connected in parallel with the above loads making the new overall power factor 0.9 lagging.

  i. Determine the reactive power absorbed by this capacitor and the value of the capacitance in Farads. [4pts]

  ii. Calculate the new rms source current, what do you notice? [2pts]

a. Load B: $P_B = P_{app,3} \cos\theta_B$

$= 28 \times 0.707 = 19.796\ kw$

$Q_B = \sqrt{P_{app,B}^2 - P_B^2} = \sqrt{28^2 - (19.796)^2} = 19.8\ kVAR$

Load C: $P_C = 2\ kW$ $\quad Q_C = -2\ kVAR$ (capacitive)

$P_{tot} = P_A + P_B + P_C = 4 + 19.796 + 2 = 25.796\ kw$

$Q_{tot} = Q_A + Q_B + Q_C = 0 + 19.8 - 2 = 17.8\ kVAR$

$\tan\theta_{tot} = \dfrac{Q_{tot}}{P_{tot}} = 0.69 \Rightarrow \theta_{tot} = 34.6 \Rightarrow \cos\theta_{tot} = 0.823$

$$P_{source} = P_{total} = V_{rms} I_{rms} \cos\theta_{tot}$$

$$\Rightarrow I_{rms} = \frac{25.796}{480 \times 0.823} = 0.0652 \text{ kA} = 65.2 \text{ A}$$

b. $\cos\theta_t = 0.9 \longrightarrow \theta_t = +25.8$
↳ lagging

P_{tot} is the same $\longrightarrow P_{tot} = 25.796$ kW

New $Q_{tot} \longrightarrow Q_{tot,new} = P_{tot} \tan\theta_t = 25.796 \times \tan 25.8$

$$= 25.796 \times 0.484$$

$$= 12.49 \text{ kVAR}$$

$$Q_{tot,new} = Q_{tot,old} + Q_C$$

$$\Rightarrow Q_C = 12.49 - 17.8 = -5.35 \text{ kVAR}$$

$$Q_C = -\frac{V_{rms}^2}{X_C} \Rightarrow X_C = \frac{-V_{rms}^2}{Q_C} = \frac{480 \times 480}{5.35 \times 10^3} = 43\,\Omega$$

$$\Rightarrow \frac{1}{C\omega} = 43 \Rightarrow C = \frac{1}{43 \times 2\pi \times 60} = 61.68\ \mu F$$

(c) $P_{tot} = V_{rms} I_{rms} \cos\theta_t$

$$\Rightarrow I_{rms} = \frac{25.796}{480 \times 0.9} = 59.7 \text{ A}$$ → Lower Current → better losses